

## SPECIFICATION

POSITIONING DATA CALCULATING PROCEDURE, POSITIONING DATA  
CALCULATING APPARATUS AND ARTICULATED MANIPULATOR

5

BACKGROUND OF THE INVENTIONField of the Invention

The present invention relates to an articulated manipulator comprising a plurality of links, and joints  
10 rotatably connecting the links. More particularly, the present invention relates to an articulated manipulator comprising a plurality of links, and coaxial joints each for connecting the two adjacent links for coaxial rotation, and diagonal joints each for connecting the  
15 two adjacent links such that one of the two adjacent links makes a conical revolution relative to the other.

Description of the Related Art

Prior art related to articulated manipulators disclosed in, for example, JP62-148182A, JP56-163624A  
20 and JP2001-138279A has a diagonal joint connecting two links. The diagonal joint connects the two links such that one of the two links makes a conical revolution on the diagonal joint. Each of the two adjacent links connected by the diagonal joint revolves about a  
25 rotation axis inclined at a predetermined angle to its axis.

For example, an articulated manipulator comprises a plurality of links arranged in series, and a plurality of joints rotatably connecting the adjacent links,  
30 respectively. The joints include coaxial joints and diagonal joints. The coaxial joint connects the adjacent links so that the adjacent links are able to turn about a rotation axis aligned with their axes. The diagonal joint connects the adjacent links such that one of the  
35 two adjacent links is able to make a conical revolution relative to the other about a rotation axis inclined at

an angle to its axis.

The articulated manipulator moves a terminal device connected to its terminal end along a predetermined path. The motion of the terminal device needs to be converted  
5 into the respective turning motions of the links to move the terminal device by the articulated manipulator; that is, the rotation angles of the links must be determined by inverse transformation on the basis of a position at which the terminal device is to be positioned and an  
10 orientation in which the terminal device is to be set.

According to the prior art disclosed in JP56-163624A, the inverse transformation is impossible unless the articulated manipulator has seven joints connecting the links, namely, a coaxial joint, a diagonal joint, a  
15 coaxial joint, a diagonal joint, a coaxial joint, a diagonal joint and a coaxial joint arranged in that order. In the case of using this prior art, analytical solution cannot be obtained through inverse transformation for an articulated manipulator having  
20 less than seven joints. Therefore, there has been no articulated manipulator having six or less joints.

If there is an articulated manipulator having six joints, complicated simultaneous equations must be solved for inverse transformation. It is difficult to  
25 obtain analytical solutions even if a computer capable of executing a formula manipulation program is used for solving the complicated simultaneous equations. Therefore, the rotation angles of the links of an articulated manipulator having six or less joints must  
30 be determined by a convergence operation, such as an operation using the Newton-Raphson formula. The rotation angles of the links cannot be determined quickly by the convergence operation.

Technique disclosed in JP2001-138279A determines  
35 rotation angles of the links by an interpolatory approximation method. This method determines a

representative point and orientation of the terminal end of the articulated manipulator by an analytical direct transformation after the rotation angles of the links are determined, and stores rotation angles of the links with the terminal device located at a plurality of representative points in a database beforehand. Inverse transformation determines rotation angles of the links by interpolatory approximation using data included in the database. The terminal end of an articulated manipulator having less than seven joints cannot accurately be positioned and oriented by operating the articulated manipulator on the basis of this method because the rotation angles are approximate values.

Fig. 10 shows a part of an articulated manipulator 1 whose location is controlled by interpolatory approximation. Generally, the articulated manipulator 1 stores rotation angles of its links when a terminal device 10 is located at representative points. For example, to move the terminal device 10 from a first representative point A to a second representative point B, the articulated manipulator 1 reads the rotation angles of the links with the terminal device 10 at the representative point A, and those of the links with the terminal device 10 at the representative point B from the database. Then, the articulated manipulator 1 carries out interpolatory calculations using the rotation angles of the links with the terminal device 10 at the representative point A and those of the same with the terminal device 10 at the representative point B to determine interpolated rotation angles. The links are turned to the interpolated rotation angles to shift the terminal device 10 from the representative point A to the representative point B.

Since the links are turned to the rotation angles determined by interpolatory approximation, the terminal device 10 cannot accurately be located; that is, the

terminal device 10 cannot smoothly moved along a prescribed path.

There are a plurality of combinations of the rotation angles of the links to set the terminal device 10 in a desired orientation at a desired position. However, when the rotation angles of the links are determined by a convergence operation or interpolatory approximation, only a single combination of rotation angles can be determined and hence an optimum combination of rotation angles of the links cannot be determined.

The rotation angles of the links of the articulated manipulator having six or less joints cannot analytically be determined in locating the terminal device 10 in a desired orientation at a desired position and, consequently, the terminal device 10 cannot quickly and accurately be located in the desired orientation at the desired position. Moreover, a plurality of combinations of the rotation angles of the links cannot be determined.

#### SUMMARY OF THE INVENTION

Accordingly, it is an object of the present invention to provide a positioning data calculating procedure and a positioning data calculating apparatus capable of analytically determining angles of the links of an articulated manipulator to locate an object in a desired orientation at a desired position.

According to a first aspect of the present invention, a positioning data calculating procedure for calculating relative rotation angles for first to sixth links arranged in series and included in an articulated manipulator including coaxial joints each connecting the adjacent ones of the first to the sixth link so that the adjacent links are able to turn about a rotation axis aligned with their axes, and diagonal joints each

connecting the adjacent ones of the first to the sixth link so that the adjacent links are able to revolve about a rotation axis inclined to their axes, in which the sixth link is attached rotatably to a predetermined object, and a rotation axis about which the sixth link and the object turn, a rotation axis about which the fifth and the sixth link turn and a rotation axis about which the fourth and the fifth link turn intersect each other at a triaxial intersection point comprises: a data entering step of entering data on position and orientation of the object in a reference coordinate system with an x-axis, a y-axis and a z-axis orthogonally intersecting each other and fixed relative to the first link; a triaxial intersection point coordinate calculating step of calculating x-, y- and z-coordinate of the triaxial intersection point in the reference coordinate system on the basis of the entered data on the position and orientation of the object; a first-to-third rotation angle calculating step of calculating first to third rotation angles by solving coordinate expressions including an x-coordinate expression representing the x-coordinate of the triaxial intersection point, a yz addition coordinate expression representing the sum of the y- and the z-coordinate of the triaxial intersection point, and a yz subtraction coordinate expression representing the remainder of subtraction of the z-coordinate from the y-coordinate of the triaxial intersection point, and including a first rotation angle corresponding to a rotation angle through which the second link is turned relative to the first link, a second rotation angle corresponding to a rotation angle through which the third link is turned relative to the second link, and a third rotation angle corresponding to a rotation angle through which the fourth link is turned relative to the third link as variables; and a fourth-to-sixth angle calculating step

of calculating a fourth rotation angle through which the fifth link is turned relative to the fourth link, a fifth rotation angle through which the sixth link is turned relative to the fifth link, and a sixth rotation  
5 angle through which the object is turned relative to the sixth link on the basis of the first to the third rotation angle and the orientation of the object.

According to the present invention, the articulated manipulator as an object of orientation control has the  
10 triaxial intersection point where the rotation axis about which the object and the sixth link turn, the rotation axis about which the sixth and the fifth link turn, and the rotation axis about which the fifth and the fourth link turn are coincident. Therefore the  
15 coordinates of the triaxial intersection point can uniquely be determined when the position and orientation of the object is determined. The object may be, for example, a base or a terminal device.

When the data on the position and orientation of  
20 the object is entered, the x-, the y- and the z-coordinate of the triaxial intersection point can be determined. The coordinate expressions including the x-coordinate expression, the yz addition coordinate expression and the yz subtraction coordinate expression  
25 are solved. The use of the yz addition coordinate expression and the yz subtraction coordinate expression enables the analytical determination of the first to the third angle included in the coordinate expressions. When the first to the third angle are determined by the  
30 first-to-third angle calculating step, the fourth to the sixth angle can analytically be determined by the fourth-to-sixth angle calculating step.

Thus, the first to the sixth angle can quickly and accurately be determined through the analytical solution  
35 of the expressions. All the possible combinations of the angles can be obtained in moving the object to a desired

position.

According to a second aspect of the present invention, a rotation angle calculating procedure for calculating relative rotation angles for first to sixth  
5 links arranged in series and included in an articulated manipulator including coaxial joints each connecting the adjacent ones of the first to the sixth link so that the adjacent links are able to turn about a rotation axis aligned with their axes, and diagonal joints each  
10 connecting the adjacent ones of the first to the sixth link so that the adjacent links are able to revolve about a rotation axis inclined to their axes, in which the sixth link is attached rotatably to a predetermined object, and a rotation axis about which the fifth and  
15 the sixth link turn, and a rotation axis about which the fourth and the fifth link turn intersect each other at a biaxial intersection point comprises: a data entering step of entering data on a position and orientation of the object in a reference coordinate system with an x-axis, a y-axis and a z-axis orthogonally intersecting  
20 each other and fixed relative to the first link; a biaxial intersection point coordinate calculating step of calculating x-, y- and z-coordinate of the biaxial intersection point in the reference coordinate system on the basis of the entered data on the position and  
25 orientation of the object; a first-to-third angle calculating step of calculating first to third angles by solving coordinate expressions including an x-coordinate expression representing the x-coordinate of the biaxial intersection point, a yz addition coordinate expression representing the sum of the y- and the z-coordinate of the biaxial intersection point, and a yz subtraction  
30 coordinate expression representing the remainder of subtraction of the z-coordinate from the y-coordinate of the biaxial intersection point, and including a first  
35 angle corresponding to a rotation angle through which

the second link is turned relative to the first link, a second angle corresponding to a rotation angle through which the third link is turned relative to the second link, and a third angle corresponding to a rotation angle through which the fourth link is turned relative to the third link as variables; and a fourth-and-fifth angle calculating step of calculating a fourth rotation angle through which the fifth link is turned relative to the fourth link, and a fifth rotation angle through which the sixth link is turned relative to the fifth link, on the basis of the first to the third rotation angle and the orientation of the object.

According to the present invention, the articulated manipulator as an object of orientation control has the biaxial intersection point where the rotation axis about which the sixth and the fifth link turn, and the rotation axis about which the fifth and the fourth turn are coincident. Therefore the coordinates of the biaxial intersection point can uniquely be determined when the position and orientation of the object is determined. The object may be, for example, a base or a terminal device.

When the data on the position and orientation of the object is entered, the x-, the y- and the z-coordinate of the biaxial intersection point can be determined. The coordinate expressions including the x-coordinate expression, the yz addition coordinate expression and the yz subtraction coordinate expression are solved. The use of the yz addition coordinate expression and the yz subtraction coordinate expression enables the analytical determination of the first to the third angle included in the coordinate expressions. When the first to the third angle are determined by the first-to-third angle calculating step, the fourth and the fifth angle can analytically be determined by the fourth-and-fifth angle calculating step.



Thus, the first to the fifth angle can quickly and accurately be determined through the analytical solution of the expressions. All the possible combinations of the angles can be obtained to move the object to a desired position.

According to a third aspect of the present invention, a positioning data calculating apparatus for calculating relative rotation angles of first to sixth links arranged in series and included in an articulated manipulator including coaxial joints each connecting the adjacent ones of the first to the sixth link so that the adjacent links are able to turn about a rotation axis aligned with their axes, and diagonal joints each connecting the adjacent ones of the first to the sixth link so that the adjacent links are able to revolve about a rotation axis inclined to their axes, in which the sixth link is attached rotatably to a predetermined object, and a rotation axis about which the sixth link and the object turn, a rotation axis about which the fifth and the sixth link turn and a rotation axis about which the fourth and the fifth link turn intersect each other at a triaxial intersection point comprises: a data entering means for entering data on a position and orientation of the object in a reference coordinate system with an x-axis, a y-axis and a z-axis orthogonally intersecting each other and fixed relative to the first link; a calculating means for calculating an x-, a y- and z-coordinate of the triaxial intersection point in the reference coordinate system on the basis of the entered data on the position and orientation of the object, determining first to third rotation angles by solving coordinate expressions including an x-coordinate expression representing the x-coordinate of the triaxial intersection point, a yz addition coordinate expression representing the sum of the y- and the z-coordinate of the triaxial intersection

point, and a yz subtraction coordinate expression representing the remainder of subtraction of the z-coordinate from the y-coordinate of the triaxial intersection point, and including a first angle  
5 corresponding to a rotation angle through which the second link is turned relative to the first link, a second angle corresponding to a rotation angle through which the third link is turned relative to the second link, and a third angle corresponding to a rotation  
10 angle through which the fourth link is turned relative to the third link as variables, and calculating a fourth rotation angle through which the fifth link is turned relative to the fourth link, a fifth rotation angle through which the sixth link is turned relative to the  
15 fifth link, and a sixth rotation angle through which the object is turned relative to the sixth link on the basis of the first to the third rotation angle and the orientation of the object; and an output means for providing calculated data calculated by the calculating  
20 means.

According to the present invention, the articulated manipulator as an object of orientation control has the triaxial intersection point where the rotation axis about which the object and the sixth link turn, the  
25 rotation axis about which the sixth and the fifth link turn, and the rotation axis about which the fifth and the fourth link turn are coincident. Therefore the coordinates of the triaxial intersection point can uniquely be determined when the position and orientation  
30 of the object is determined. The object may be, for example, a base or a terminal device.

When the data on the position and orientation of the object is entered by the data entering means, the x-, the y- and the z-coordinate of the triaxial intersection  
35 point can be determined. The calculating means solves the coordinate expressions including the x-coordinate

expression, the yz addition coordinate expression and the yz subtraction coordinate expression. The first to the third angle included in the coordinate expressions can analytically be determined by using the yz addition  
5 coordinate expression and the yz subtraction coordinate expression. When the first to the third angle are determined, the fourth to the sixth angle can analytically be determined on the basis of the first to the third angle. The calculating means gives the first  
10 to the sixth angle to the output means, and the output means delivers the calculated results.

Thus, the first to the sixth angle can quickly and accurately be determined through the analytical solution of the expressions in locating the object in a desired  
15 orientation at a desired position. All the possible combinations of the angles can be obtained in moving the object to a desired position.

According to a fourth aspect of the present invention, a positioning data calculating apparatus for  
20 calculating relative rotation angles for first to sixth links arranged in series and included in an articulated manipulator including coaxial joints each connecting the adjacent ones of the first to the sixth link so that the adjacent links are able to turn about a rotation axis  
25 aligned with their axes, and diagonal joints each connecting the adjacent ones of the first to the sixth link so that the adjacent links are able to revolve about a rotation axis inclined to their axes, in which the sixth link is attached rotatably to a predetermined  
30 object, and a rotation axis about which the fifth link and the sixth link turn, and a rotation axis about which the fourth and the fifth link turn intersect each other at a biaxial intersection point comprise: a data entering means for entering data on a position and  
35 orientation of the object in a reference coordinate system with an x-axis, a y-axis and a z-axis

orthogonally intersecting each other and fixed relative to the first link; a calculating means for calculating x-, y- and z-coordinate of the biaxial intersection point in the reference coordinate system on the basis of  
5 the entered data on the position and orientation of the object, determining first to third rotation angles by solving coordinate expressions including an x-coordinate expression representing the x-coordinate of the biaxial intersection point, a yz addition coordinate expression  
10 representing the sum of the y- and the z-coordinate of the biaxial intersection point, and a yz subtraction coordinate expression representing the remainder of subtraction of the z-coordinate from the y-coordinate of the biaxial intersection point, and including a first  
15 angle corresponding to a rotation angle through which the second link is turned relative to the first link, a second angle corresponding to a rotation angle through which the third link is turned relative to the second link, and a third angle corresponding to a rotation  
20 angle through which the fourth link is turned relative to the third link as variables, and calculating a fourth rotation angle through which the fifth link is turned relative to the fourth link, and a fifth rotation angle through which the sixth link is turned relative to the  
25 fifth link on the basis of the first to the third rotation angle and the orientation of the object; and an output means for providing calculated data calculated by the calculating means.

According to the present invention, the articulated  
30 manipulator as an object of orientation control has the biaxial intersection point where the rotation axis about which the sixth link and the fifth link turn, and the rotation axis about which the fifth and the fourth link turn are coincident. Therefore the coordinates of the  
35 biaxial intersection point can uniquely be determined when the position and orientation of the object is

determined. The object may be, for example, a base or a terminal device.

When the data on the position and orientation of the object is entered by the data entering means, the x-,  
5 the y- and the z-coordinate of the biaxial intersection point can be determined by the calculating means. The calculating means solves the coordinate expressions including the x-coordinate expression, the yz addition coordinate expression and the yz subtraction coordinate  
10 expression. The first to the third angle included in the coordinate expressions can analytically be determined by using the yz addition coordinate expression and the yz subtraction coordinate expression. When the first to the third angle are determined, the fourth and the fifth  
15 angle can analytically be determined on the basis of the first to the third angle. The calculating means gives the first to the fifth angle to the output means, and the output means provides the calculated results.

Thus, the first to the fifth angle can quickly and  
20 accurately be determined through the analytical solution of the expressions in locating the object in a desired orientation at a desired position. All the possible combinations of the angles can be obtained to move the object to a desired position.

25 According to a fifth aspect of the present invention, an articulated manipulator comprises: first to sixth links arranged in series; coaxial joints each connecting the adjacent ones of the first to the sixth link so that the adjacent links are able to turn about a  
30 rotation axis aligned with their axes; and diagonal joints each connecting the adjacent ones of the first to the sixth link so that the adjacent links are able to revolve about a rotation axis inclined to their axes;

wherein the sixth link is attached rotatably to a  
35 predetermined object so that the object is able to turn about the axis of the sixth link, the fifth and the

sixth link are connected by the diagonal joint, the fourth and the fifth link are connected by the coaxial joint, the third and the fourth link are connected by the diagonal joint, the second and the third link are  
5 connected by the coaxial joint, and the first and the second link are connected by the diagonal joint.

According to the present invention, a rotation axis about which the object and the sixth link turn, a rotation axis about which the sixth and the fifth link  
10 turn, and a rotation axis about which the fifth and the fourth link turn are coincident at a triaxial intersection point. Therefore the coordinates of the triaxial intersection point can uniquely be determined by an orientation controller when the position and  
15 orientation of the object is determined. The orientation controller is able to determine analytically the first to the sixth angle for the links for locating the object in a desired orientation at a desired position.

When at least the rotation axis about which the two  
20 links connected by the diagonal joint among the joints connecting the first to the third link turn inclines at an angle of  $45^\circ$  to the axes of the two links connected by the diagonal joint, the orientation controller is able to determine the angle by using simple expressions. Thus,  
25 the orientation controller is able to determine the first to the sixth angle in a shorter time.

According to a sixth aspect of the present invention, an articulated manipulator comprises: first to sixth links arranged in series; coaxial joints each  
30 connecting the adjacent ones of the first to the sixth link so that the adjacent links are able to turn about a rotation axis aligned with their axes; and diagonal joints each connecting the adjacent ones of the first to the sixth link so that the adjacent links are able to  
35 revolve about a rotation axis inclined to their axes;

wherein the sixth link is fixedly connected to a

predetermined object, the fifth and the sixth link are connected by the diagonal joint, the fourth and the fifth link are connected by the coaxial joint, the third and the fourth link are connected by the diagonal joint, the second and the third link are connected by the coaxial joint, and the first and the second link are connected by the diagonal joint.

According to the present invention, a rotation axis about which the fifth and the sixth link turn, and a rotation axis about which the fifth and the fourth link turn are coincident at a biaxial intersection point. Therefore the coordinates of the biaxial intersection point can uniquely be determined by an orientation controller when the position and orientation of the object is determined. The orientation controller is able to determine analytically the first to the fifth angle for the links for locating the object in a desired orientation at a desired position.

When at least the rotation axis about which the two links connected by the diagonal joint among the joints connecting the first to the third link turn inclines at an angle of  $45^\circ$  to the axes of the two links connected by the diagonal joint, the orientation controller is able to determine the angle by using simple expressions. Thus, the orientation controller is able to determine the first to the sixth angle in a shorter time.

#### BRIEF DESCRIPTION OF THE DRAWINGS

The above and other objects, features and advantages of the present invention will become more apparent from the following description taken in connection with the accompanying drawings, in which:

Fig. 1 is a schematic front elevation of an articulated manipulator in a first embodiment according to the present invention;

Figs. 2(1) to 2(8) are views of the articulated

manipulator shown in Fig. 1 in different shapes, respectively, formed by orientation control operations executed by a positioning arithmetic unit;

Fig. 3 is diagrammatic view of assistance in explaining an orientation control operation to be carried out by the positioning arithmetic unit to control the articulated manipulator in moving a terminal device;

Fig. 4 is a flow chart of a positioning data calculating procedure to be executed by the positioning arithmetic unit;

Fig. 5 is a flow chart of a positioning data calculating procedure in a preferred embodiment according to the present invention;

Fig. 6 is a typical view of a true model of an articulated manipulator according to the present invention;

Fig. 7 is a typical view of an inverted model in inverse relation to the true model;

Fig. 8 is a typical view of an equivalent model formed by replacing one of the joints of an articulated manipulator represented by the inverted model shown in Fig. 7 with an equivalent joint;

Fig. 9 is a typical view of an articulated manipulator in a second embodiment according to the present invention; and

Fig. 10 is a diagrammatic view of assistance in explaining an orientation control operation for controlling the orientation of a conventional articulated manipulator.

#### DESCRIPTION OF THE PREFERRED EMBODIMENTS

Referring to Fig. 1, an articulated manipulator in a first embodiment according to the present invention has an arm assembly formed by connecting a plurality of links in series by joints. The joints include coaxial



joints and diagonal joints. Each coaxial joint connects the adjacent links so as to be turnable about a rotation axis aligned with the respective axes of the adjacent links. Each diagonal joint connects the two adjacent  
5 links such that the two adjacent links are able to make a conical revolution about a rotation axis inclined to the respective axes of the two adjacent links. In this specification, the term "conical revolution" signifies the revolution of one of two adjacent links connected by  
10 a joint on the joint about a rotation axis relative to the other on an imaginary cone having its tip on the joint, and the term "rotation" signifies angular displacements including those through angles not greater than  $360^\circ$ . The articulated manipulator thus formed by  
15 assembling the links, the coaxial joints and the diagonal joints is capable of positioning a terminal device in a three-dimensional space by turning the links so that the articulated manipulator twists and turns like a snake. Thus, the articulated manipulator of the  
20 present invention is capable of satisfactorily operating in complicated facilities in which the articulated manipulator needs to operate along complicated work paths.

The articulated manipulator of the present  
25 invention is used as an industrial robot. A desired terminal device, i.e., an end effector, can be attached to the free end of the articulated manipulator, the terminal device can be positioned at a desired position, and can operate for necessary work in a narrow space.  
30 The articulated manipulator can be used in many tasks, such as material handling, sealing, painting, arc welding and such.

Referring to Fig. 1, an articulated manipulator 20  
in a first embodiment according to the present invention  
35 is a six-degree of freedom articulated manipulator including six links, namely, a first link  $c_1$ , a second

link c2, a third link c3, a fourth link c4, a fifth link c5 and a sixth link c6, and six joints, namely, a first joint d1, a second joint d2, a third joint d3, a fourth joint d4, a fifth joint d5 and a sixth joint d6, connecting the adjacent links. The joints d1 to d6 connects the links c1 to c6 to form an arm assembly. The three joints d1, d3 and d5 are coaxial joints, and the three joints d2, d4 and d6 are diagonal joints. Each of the diagonal joints d2, d4 and d6 connects the adjacent links such that the two adjacent links are able to revolve relative to each other about a rotation axis inclined at an angle of  $45^\circ$  to the respective axes of the adjacent links.

The links c1 to c6 are arranged in series in that order from a base end outward. A terminal device 22 is connected to the free end of the sixth link c6. The links c1 to c6 can coaxially be extended in a straight structure as shown in Fig. 1.

The coaxial joint d1 connects an inner end 23 of the first link c1 to a base 21. The base 21 may fixedly be installed at a predetermined position or may be movable. The first link c1 is able to rotate about a rotation axis L1 coaxial with its axis relative to the base 21. The diagonal joint d2 connects an outer end 24 of the first link c1 to the second link c2.

The second link c2 is able to make a conical revolution relative to the first link c1 on the diagonal joint d2 about an inclined rotation axis L2 inclined at an angle of  $45^\circ$  to the axis of the second link c2. The coaxial joint d3 connects an outer end 25 of the second link c2 opposite the first link c1 to the third link c3.

The second link c2 and the third link c3 are able to turn about a rotation axis L3 aligned with their axes. The diagonal joint d4 connects an outer end 26 of the third link c3 opposite the second link c2 to the fourth link c4.

The third link c3 and the fourth link c4 are able to make a conical revolution relative to each other on the diagonal joint d4 about an inclined rotation axis L4 inclined at an angle of  $45^\circ$  to the respective axes of the third link c3 and the fourth link c4. The coaxial joint d5 connects an outer end 27 of the fourth link c4 opposite the third link c3 to the fifth link c5.

The fourth link c4 and the fifth link c5 are able to rotate about a rotation axis L5 aligned with the respective axes of the fourth link c4 and the fifth link c5. The diagonal joint d6 connects an outer end 28 of the fifth link c5 opposite the fourth link c4 to the sixth link c6.

The fifth link c5 and the sixth link c6 are able to make a conical revolution relative to each other on the diagonal joint d6 about an inclined rotation axis L6 inclined at an angle of  $45^\circ$  to the respective axes of the fifth link c5 and the sixth link c6. The terminal device 22 is connected to an outer end of the sixth link c6 opposite the fifth link c5. The terminal device 22 may be any suitable device, such as a material handling device capable of gripping parts.

The coaxial joints and the diagonal joints are arranged alternately in the arm assembly formed by connecting the links c1 to c6 in series. The arm assembly can straight be extended with all the axes of the links c1 to c6 in alignment. The articulated manipulator 20 has a triaxial intersection point where the rotation axis L1 about which the base 21 and the first link c1 turn, the second rotation axis L2 about which the first link c1 and the second link c2 revolve, and the rotation axis L3 about which the second link c2 and the third link c3 turn are coincident.

Rotary motors are incorporated into the links c1 to c6 to drive the links c1 to c6 for rotation. The links c1 to c6 are hollow members respectively having bores.

Power lines, signal lines and tubes are passed through the bores of the links c1 to c6 to supply power to the rotary motors, to send control signals to the rotary motors and to supply compressed air to the terminal  
5 device 22.

A conventional rotating mechanism may be employed for turning each of the links c1 to c6 about the rotation axis. For example, the articulated manipulator 20 may be provided for turning each of the links c1 to  
10 c6 with a rotating mechanism including bearings supporting the adjacent links for turning about the rotation axis, and a hollow waving gear mechanism, such as a Harmonic drive®. The waving gear mechanism has an input member and an output member, and the input and the  
15 output member rotate relative to each other. When the waving gear mechanism is employed, one of the adjacent links is connected to the input member, and the other is connected to the output member. When the input member is driven for rotation by the rotary motor, the input and  
20 the output member rotate relative to each other to turn the adjacent links relative to each other. The adjacent links can be turned relative to each other by providing the links c1 to c6 with such rotating mechanisms. The use of the hollow waving gear mechanism (Harmonic  
25 drive®) enables the adjacent links to turn relative to each other with the power lines and signal lines extended through the links c1 to c6. The articulated manipulator 20 can be suited for use in a dusty, explosive, humid working environment by covering the  
30 articulated manipulator 20 entirely or covering the power lines and signal lines extended through the bores of the links c1 to c6 with a cover formed of a waterproof, heat-resistant, shock-resistant material.

As shown in Fig. 1, the articulated manipulator 20  
35 is provided with a positioning arithmetic unit 30 for calculating angles of the links c1 to c6 relative to

each other. The positioning arithmetic unit 30 calculates relative rotation angles  $\theta_1$  to  $\theta_6$  through which the links c1 to c6 are to be turned, respectively, to locate the terminal device 22 in a desired orientation at a desired position.

The positioning arithmetic unit 30 includes an input device 31, a calculating device 32 and an output device 33. The input device 31 gives the calculating device 32 data on a position and an orientation in which the terminal device 22 is to be located in a reference coordinate system. The calculating device 32 calculates relative rotation angles  $\theta_1$  to  $\theta_6$  through which the links c1 to c6 are to be turned to locate the terminal device 22 in the desired orientation at the desired position on the basis of the data given thereto by the input device 31 and the dimensions of the articulated manipulator 20. The output device 33 provides power signals and control signals for turning the links c1 to c6 on the basis of calculated data provided by the calculating device 32. Consequently, the links c1 to c6 are turned according to the calculated data.

The positioning arithmetic unit 30 is, for example, a computer. The input device 31 includes a pointing device, such as a keyboard or a mouse, and a teaching pendant controller to be operated by the operator for the direct control of the articulated manipulator 20 by the operator. The input device 31 may be capable of entering data on a position and orientation of the end effector provided by an external device connected to the positioning arithmetic unit 30. The output device 33 generates control signals for controlling the operation of the drive motors on the basis of the calculated data, and gives the control signals to the drive motors. The output device 33 may be, for example, a drive circuit. The positioning arithmetic unit 30 may be either built in the articulated manipulator 20 or installed outside

the articulated manipulator 20.

The calculating device 32 includes a central processing unit (CPU) that executes arithmetic programs. The arithmetic programs may be stored beforehand in the CPU or may be stored in a storage device and read by the CPU. The arithmetic programs prescribe calculating methods, which will be described later.

Figs. 2(1) to 2(8) are views of the articulated manipulator 20 in different shapes, respectively, formed by orientation control operations executed by the positioning arithmetic unit 30. The positioning arithmetic unit 30 calculates values of the rotation angles  $\theta_1$  to  $\theta_6$  through which the links c1 to c6 must be turned, respectively, to locate the terminal device in a desired orientation at a desired position by using analytical expressions. There are eight combinations of the rotation angles  $\theta_1$  to  $\theta_6$  for locating the terminal device 22 held on the six-axis articulated manipulator 20 in a desired orientation at a desired position. The arithmetic unit 30 is able to provide the eight combinations of the angles  $\theta_1$  to  $\theta_6$  simultaneously by calculating the angles  $\theta_1$  to  $\theta_6$  using the analytical expressions. The terminal device 22 can be located in a desired orientation at a desired position by bending the articulated manipulator 20 in any one the eight shapes shown in Figs. 2(1) to 2(8). One of the eight combinations of the angles  $\theta_1$  to  $\theta_6$  through which the links c1 to c6 are to be turned is selected taking into consideration energy cost and obstacles around the articulated manipulator 20.

Fig. 3 is diagrammatic view of assistance in explaining an orientation control operation to be carried out by the positioning arithmetic unit 30 to control the articulated manipulator 20 in moving the terminal device 20. When it is desired to move the terminal device 22 from a first representative point A

to a second representative point B, intermediate points U1 to U3 that is to be passed by the terminal device 22 when the terminal device 22 moves from the first representative point A to the second representative point B are determined. Rotation angles through which the links c1 to c6 are to be turned to locate the terminal device 22 at the intermediate points U1 to U3 are calculated to move the terminal device 22 smoothly along a specified path including the intermediate points U1 to U3.

The positioning arithmetic unit 30 does not need to perform the convergence operation, which must be performed by the conventional articulated manipulator, to determine the rotation angles for the links c1 to c6. Therefore, the rotation angles can quickly be determined, the calculating device 32 does not need to have a high data processing ability, and the rotation angles for the links can be calculated even during the movement of the terminal device 22. The rotation angles for the links c1 to c6 at the representative points A and B do not need to be stored in a database, the storage capacity of the CPU may be small.

Referring to Fig. 4 showing a flow chart of a positioning data calculating procedure to be executed by the positioning arithmetic unit 30, data on the articulated manipulator 20 to be controlled including the dimensions of the links, and the types and characteristics of the joints is given in step s1. In step s2, data on the position and orientation of an object to be moved by the articulated manipulator 20 is acquired. In this case, the object is the terminal device 22 attached to the free end of the sixth link c6, and the data represents the orientation and position of the terminal device 22 in the fixed coordinate system. Then, in step s3, the coordinates of the triaxial intersection point, where the rotation axis L1 about

which the base 21 and the first link c1 turn, the second  
 rotation axis L2 about which the first link c1 and the  
 second link c2 revolve, and the rotation axis L3 about  
 which the second link c2 and the third link c3 turn are  
 5 coincident, are determined. The coordinates of the  
 triaxial intersection point are uniquely dependent on  
 the orientation and the position of the object.  
 Therefore, the coordinates of the triaxial intersection  
 point can be determined on the basis of the orientation  
 10 and position of the object determined in step s2. Then,  
 in step s4, coordinate expressions expressing the  
 coordinates of the triaxial intersection point are  
 produced using, as variables, the first to the third  
 angle, i.e., relative rotation angles of turning of the  
 15 three links about the three rotation axes other than the  
 three rotation axes intersecting at the triaxial  
 intersection point. The coordinate expressions are an x-,  
 a y- and a z-coordinate expression respectively  
 expressing the x-, the y- and the z-coordinate of the  
 20 triaxial intersection point. Then, a yz addition  
 coordinate expression representing the sum of the y- and  
 the z-coordinate of the triaxial intersection point, and  
 a yz subtraction coordinate expression representing the  
 remainder of subtraction of the z-coordinate from the y-  
 25 coordinate of the triaxial intersection point are  
 produced. Then, in step s5, the first to the third  
 angles are calculated by using the x-coordinate  
 expression, the yz addition coordinate expression and  
 the yz subtraction coordinate expression produced in  
 30 step s4. Then, the fourth to the sixth angle are  
 determined on the basis of the first to the third angle.  
 After the rotation angles for the links c1 to c6 have  
 been determined, the positioning data calculating  
 procedure is ended in step s6.

35 To facilitate the determination of the rotation  
 angles, suppose that the articulated manipulator 20



shown in Fig. 1 is a true model, an inverted model is in inverse relation to the true model with respect to the z-axis, and an equivalent model is equivalent to the inverted model and has joints including one joint different from that of the inverted model. The rotation angles for the links can more easily be determined after transforming the true model into the inverted model and the equivalent model.

Referring to Fig. 5 showing a flow chart of a positioning data calculating procedure for calculating positioning data for the positioning control of the articulated manipulator 20 shown in Fig. 1, data, on the articulated manipulator 20 as the true model to be controlled, including the dimensions of the links, and the types and characteristics of the joints necessary for calculating rotation angles for the links is given in step s11. In step s12, data on the position and orientation of the terminal device with respect to the base 21 of the true model is acquired. In step s13, a coordinate system and a position for the true model are transformed into those for the inverted model. The position and orientation of the base 21 with respect to the terminal device 22 of the inverted model are calculated. Then, in step s14, the position of the triaxial intersection point in the inverted model is determined on the basis of the position and orientation of the base 21 in the inverted model. In step s15, coordinate expressions expressing the coordinates of the triaxial intersection point in the equivalent model are produced using, as variables, the first to the third angle, i.e., relative rotation angles of turning of the three links about the three rotation axes other than the three rotation axes intersecting at the triaxial intersection point. More concretely, an x-, a y- and a z-coordinate expression respectively expressing the x-, the y- and the z-coordinate of the triaxial intersection

point in the equivalent model are produced. Then a yz addition coordinate expression representing the sum of the y- and the z-coordinate of the triaxial intersection point, and a yz subtraction coordinate expression  
 5 representing the remainder of subtraction of the z-coordinate from the y-coordinate of the triaxial intersection point are produced. Then, in step s16, the first to the third angles for the equivalent model are calculated on the basis of the position of the triaxial  
 10 intersection point in the inverted model determined in step s14, and the x-coordinate expression, the yz addition coordinate expression and the yz subtraction coordinate expression produced in step s15. Then, in step s17, the first to the third angles for the  
 15 equivalent model determined in step s16 are converted into the first to the third angle for the inverted model. Then, in step s18, the fourth to the sixth angle are determined on the basis of the first to the third angle. After converting the first to the sixth angle for the  
 20 inverted model determined in steps s17 and s18 into the first to the sixth angle for the true model, the positioning data calculating procedure is ended in step s20.

Fig. 6 shows a true model modeling the articulated  
 25 manipulator 20 of the present invention. The articulated manipulator 20 has a reference coordinate system  $\Sigma_{rb}$  fixedly attached to the base 21. The reference coordinate system  $\Sigma_{rb}$  has a reference x-, a reference y- and a reference z-axis orthogonally intersecting each  
 30 other at an origin.

Body-attached coordinate systems are fixedly attached to the links c1 to c6 and the terminal device 22, respectively. The body-attached coordinate systems are three-axis orthogonal coordinate system having their  
 35 origins at the representative points of the links c1 to c6 and the terminal device 22, respectively. Each of the

body-attached coordinate systems has a body-attached x-, a body-attached y- and a body-attached z-axis orthogonally intersecting each other and coincident at the origin.

5           In the true model shown in Fig. 6, the origin of the reference coordinate system  $\Sigma_{rb}$  is coincident with a base representative point S on an imaginary line aligned with the rotation axis L1 about which the first link c1 and the base 21 turn relative to each other. Since the  
10 terminal device 22 is fixed to the sixth link c6, the determination of the position and orientation of the terminal device 22 is equivalent to the determination of the position and orientation of the sixth link c6. The position of the terminal device 22 is represented by the  
15 coordinates of the representative point T of the sixth link c6 on an imaginary line aligned with the rotation axis L5 about which the fourth link c4 and the fifth link c5 turn relative to each other.

20           A terminal coordinate system  $\Sigma_{rt}$ , namely, a body-attached coordinate system attached to the terminal device 22, is a three-axis orthogonal coordinate system having its origin coincident with the representative point T of the sixth link c6. The representative point T of the sixth link c6 move in the reference coordinate  
25 system  $\Sigma_{rb}$  as the links c1 to c6 turn relative to each other.

          The position of the representative point T is represented by coordinates  $(T_x, T_y, T_z)$  in the reference coordinate system  $\Sigma_{rb}$ .  $T_x$  is a reference x-coordinate,  
30  $T_y$  is a reference y-coordinate, and  $T_z$  is a reference z-coordinate in the reference coordinate system  $\Sigma_{rb}$ . When the links c1 to c6 connected in series and forming a straight arm assembly are turned relative to each other, the orientation of the sixth link changes and,  
35 consequently, the terminal coordinate system  $\Sigma_{rt}$  turns relative to the reference coordinate system  $\Sigma_{rb}$ . The

orientation of the sixth link  $c_6$  is represented by orientation  $(q_0, q_a, q_t)$ , where  $q_0$ ,  $q_a$  and  $q_t$  are angles through which the terminal coordinate system  $\Sigma_{bt}$  is turned about the reference  $x$ -,  $y$ - and  $z$ -axis, respectively.

Suppose that the links  $c_1$  to  $c_6$  are in a reference state when the links  $c_1$  to  $c_6$  are extended in the straight arrangement, and the first link  $c_1$ , the second link  $c_2$ , the third link  $c_3$ , the fourth link  $c_4$ , the fifth link  $c_5$  and the sixth link  $c_6$  in the reference state are turned through a first rotation angle  $\theta_1$ , a second rotation angle  $\theta_2$ , a third rotation angle  $\theta_3$ , a fourth rotation angle  $\theta_4$ , a fifth rotation angle  $\theta_5$  and a sixth rotation angle  $\theta_6$  relative to the base  $2_1$ , the first link  $c_1$ , the second link  $c_3$ , the fourth link  $c_4$  and the fifth link  $c_5$ , respectively.

Fig. 7 is a typical view of an inverted model representing an inverted articulated manipulator  $20A$  in inverse relation to the true model. Fig. 8 is a typical view of an equivalent model representing an equivalent articulated manipulator  $20B$  formed by replacing one of the joints of the articulated inverted manipulator  $20A$  represented by the inverted model shown in Fig. 7 with an equivalent joint. Parts of the models shown in Figs. 7 and 8 will be designated by designations different from those of the parts of true model and denoted by reference characters different from those indicating the parts of the true model to facilitate understanding.

The respective first links  $e_1$ , the second links  $e_2$ , the third links  $e_3$ , the fourth links  $e_4$ , the fifth links  $e_5$  and the sixth links  $e_6$  of the inverted and the equivalent model correspond to the sixth link  $c_1$ , the fifth link  $c_5$ , the fourth link  $c_4$ , the third link  $c_3$ , the second link  $c_2$  and the first link  $c_1$  of the true model, respectively. Thus, the first links  $e_1$  to the sixth links  $e_6$  of the inverted and the equivalent model

correspond to the sixth link  $c_6$  to the first link  $c_1$  of the true model, respectively.

Similarly, joints of the inverted model, rotation angles through which the links  $e_1$  to  $e_6$  of the inverted model are turned, and longitudinal sizes of the links  $e_1$  to  $e_6$  of the inverted model are denoted by reference characters  $f_1$  to  $f_6$ ,  $q_1$  to  $q_6$ , and  $i_0$  to  $i_6$  different from those denoting the corresponding joints, rotation angles and longitudinal sizes of the true model, respectively.

The equivalent articulated manipulator 20B represented by the equivalent model differs from the inverted articulated manipulator 20A represented by the inverted model in a joint F3 connecting the third link  $e_3$  and the fourth link  $e_4$ . The joint F3 of the equivalent articulated manipulator 20B connects the third link  $e_3$  and the fourth link  $e_4$  such that the respective axes of the third link  $e_3$  and the fourth link  $e_4$  are able to turn about a rotation axis  $L_3$  perpendicular thereto. Thus, the third link  $e_3$  and the fourth link  $e_4$  turn on the joint F3 in a plane including the axes thereof.

In the inverted model, a reference coordinate system  $\Sigma_{sb}$  has its origin at the representative point T of the first link  $e_1$  coincident with the representative point T of the sixth link  $c_6$  of the true model. In the inverted model, a base coordinate system  $\Sigma_{st}$ , i.e., a body-attached coordinate system fixed to the base 21, has its origin coincident with the base representative point S. When the links  $e_1$  to  $e_6$  of the inverted model are extended coaxially in a straight arm assembly, the body x-, the body y- and the body z-axis are parallel to the reference x-, the reference y- and the reference z-axis of the reference coordinate system  $\Sigma_{sb}$ , respectively. When the links  $e_1$  to  $e_6$  are extended coaxially in a straight arm assembly, the axes of the

links  $e_1$  to  $e_6$  are parallel to the reference  $z$ -axis of the reference coordinate system  $\Sigma_{sb}$  and the base  $z$ -axis of the base coordinate system  $\Sigma_{st}$ . A direction from the origin of the reference coordinate system  $\Sigma_{sb}$  toward the origin of the base coordinate system  $\Sigma_{st}$  is a positive direction for the reference  $z$ -axis and the body  $z$ -axis.

The articulated manipulator 20 in the first embodiment executes steps  $s_1$  to  $s_6$  when the terminal device 22 is fixed and the base 21 is movable. In the inverted model, the base 21 is regarded as an object, and rotation angles  $q_1$  to  $q_6$  through which the links  $e_1$  to  $e_6$  are to be turned, respectively, to set the object in a desired orientation at a desired position are determined.

The rotation angles  $\theta_1$  to  $\theta_6$  through which the links  $c_1$  to  $c_6$  of the articulated manipulator 20 represented by the true model are turned to set the terminal device 22 in a desired orientation at a desired position can be determined by converting the rotation angles  $q_1$  to  $q_6$  determined for the links  $e_1$  to  $e_6$  of the inverted model.

Rotation angles for the links of an articulated manipulator having a triaxial intersection point on a terminal device, i.e., a movable end, can more easily be determined than those for the links of an articulated manipulator having a triaxial intersection point on a base, i.e., a fixed end. Therefore, the rotation angles  $\theta_1$  to  $\theta_6$  through which the links  $c_1$  to  $c_6$  of the articulated manipulator 20 having the triaxial intersection point on the base can easily be determined by converting the rotation angles  $q_1$  to  $q_6$  determined for the links  $e_1$  to  $e_6$  of the inverted model into those for the true model.

The positioning data calculating procedure for calculating positioning data will be described with reference to a flow chart shown in Fig. 5. Preparatory

operations for determining the rotation angles  $\theta_1$  to  $\theta_6$  for the links  $c_1$  to  $c_6$  are performed in step s11. In step s11, parameters of the articulated manipulator, and data, on the true model, including the longitudinal sizes  $H_0$  to  $H_6$  of the links  $c_1$  to  $c_6$ , angles at which the rotation axis  $L_2$  about which the first link  $c_1$  and the second link  $c_2$  are turned is inclined to the axes of the first link  $c_1$  and the second link  $c_2$ , the rotation axis  $L_4$  about which the third link  $c_3$  and the fourth link  $c_4$  are turned is inclined to the axes of the third link  $c_3$  and the fourth link  $c_4$ , and an angle at which the rotation axis  $L_6$  about which the fifth link  $c_5$  and the sixth link  $c_6$  are turned is inclined to the axes of the fifth link  $c_5$  and the sixth link  $c_6$ , respectively, is entered. In this embodiment, it is supposed that the rotation axes  $L_2$ ,  $L_4$  and  $L_6$  are inclined at the same angle  $\alpha$  to the axes of the associated links, and the angle  $\alpha = 45^\circ$ .

After the parameters of the articulated manipulator and the data on the articulated manipulator have been determined, data on the position and orientation of the terminal device 22 with respect to the reference coordinate system  $\Sigma_{rb}$  of the true model is acquired in step s12. More specifically, coordinates  $(T_x, T_y, T_z)$  of the representative point  $T$  of the sixth link  $c_6$ , and the orientation  $(q_0, q_a, q_t)$  of the sixth link  $c_6$  are acquired.

In step s13, a transformation matrix for transforming the true model into the inverted model is produced, and the orientation of the base 21 of the inverted model is determined by using the transformation matrix. The orientation of the base 21 of the inverted model is represented by angles  $q_x$ ,  $q_y$  and  $q_z$  through which the base coordinate system  $\Sigma_{st}$  is turned about the reference x-, the reference y- and the reference z-axis, respectively, of the reference coordinate system  $\Sigma_{sb}$ .

After the orientation ( $q_x$ ,  $q_y$ ,  $q_z$ ) of the base 21 has been determined, the position of the representative point S of the base 21 of the inverted model is determined. The position of the representative point S is represented by coordinates  $P_x$ ,  $P_y$  and  $P_z$  on the reference x-, the reference y- and the reference z-axis of the reference coordinate system  $\Sigma_{sb}$ .

On step s14, the position of the triaxial intersection point is determined. The triaxial intersection point is coincident with the representative point p5 of the fifth link. In the articulated manipulator 20 shown in Fig. 1, the triaxial intersection point is coincident with the intersection of the rotation axis L5 about which the fifth link e5 and the sixth link e6 are turned and the axes of the fifth link e5 and the sixth link e6, and is on the joint connecting the fifth link e5 and the sixth link e6. A transformation matrix  $R_{05}$  for transforming a reference coordinate system into a body-attached coordinate system attached to the fifth link e5 is produced. The transformation matrix  $R_{05}$  is expressed by Expression (1).

$$\begin{aligned}
 R_{01} &= Rx[\alpha] \cdot Rz[q_1] \cdot Rx[-\alpha] \\
 R_{02} &= R_{01} \cdot Rz[q_2] \\
 R_{03} &= R_{02} \cdot Rx[\alpha] \cdot Rz[q_3] \cdot Rx[-\alpha] \\
 R_{04} &= R_{03} \cdot Rz[q_4] \\
 R_{05} &= R_{04} \cdot Rx[\alpha] \cdot Rz[q_5] \cdot Rx[-\alpha] \quad \dots (1)
 \end{aligned}$$

where  $R_{01}$  to  $R_{05}$  are transformation matrices for converting the reference coordinate system into the body-attached coordinate systems, and  $\alpha$  is the angle at which the rotation axes are inclined to the axes of the associated links at the joints. The angle  $\alpha$  in matrix  $R_{01}$  is the angle at which the rotation axis L1 about which the first link e1 and the second link e2 are turned inclines to the axes of the links e1 and e2. The



angle  $\alpha$  in matrix  $R_{03}$  is the angle at which the rotation axis  $L3$  about which the third link  $e3$  and the fourth link  $e4$  are turned inclines to the axes of the links  $e3$  and  $e4$ . The angle  $\alpha$  in matrix  $R_{05}$  is the angle at which the rotation axis  $L5$  about which the fifth link  $e5$  and the sixth link  $e6$  are turned inclines to the axes of the links  $e5$  and  $e6$ . The second link  $e2$  is turned through a first rotation angle  $q_1$  relative to the first link  $e1$ , the third link  $e3$  is turned through a second rotation angle  $q_2$  relative to the second link  $e2$ , the fourth link  $e4$  is turned through a third rotation angle  $q_3$  relative to the third link  $e3$ , the fifth link  $e5$  is turned through a fourth rotation angle  $q_4$  relative to the fourth link  $e4$ , the sixth link  $e6$  is turned through a fifth rotation angle  $q_5$  relative to the fifth link  $e5$ , and the base 21 is turned through a sixth rotation angle  $q_6$  relative to the sixth link  $e6$ . When the links  $e1$  to  $e6$  are extended in a straight arm assembly, the rotation angles  $q_1$  to  $q_6$  are zero.

In Expression (1),  $R_z[0]$ ,  $R_y[0]$  and  $R_x[0]$  are rotation matrixes for turning the orientation of the body-attached coordinate system relative to the reference coordinate system. The rotation matrices  $R_x[0]$ ,  $R_y[0]$  and  $R_z[0]$  are used for turning the body-attached coordinate system through an angle  $\theta$  about the reference x-, the reference y- and the reference z-axis, respectively. Components of the rotation matrices  $R_z[0]$ ,  $R_y[0]$  and  $R_x[0]$  are expressed by Expressions (2), (3) and (4), respectively.

$$R_z[\theta] = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \dots \quad (2)$$

$$R_y[\theta] = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \quad \dots \quad (3)$$

$$R_x[\theta] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \quad \dots \quad (4)$$

where  $\theta$  is a variable and is substituted by an optional value. A matrix  $R_z[q_1]$  is obtained by substituting  $\theta$  of Expression (2) by  $q_1$ . In this specification,  $\sin \theta$  and  $\cos \theta$  represent the sine and cosine of the angle  $\theta$ , and a symbol "." indicates multiplication. The position of the representative point p5 of the fifth link e5 is expressed by Expression(5).

$$p5 = \begin{pmatrix} 0 \\ 0 \\ i0 \end{pmatrix} + R_{01} \cdot \begin{pmatrix} 0 \\ 0 \\ i1 \end{pmatrix} + R_{02} \cdot \begin{pmatrix} 0 \\ 0 \\ i2 \end{pmatrix} + R_{03} \cdot \begin{pmatrix} 0 \\ 0 \\ i3 \end{pmatrix} + R_{04} \cdot \begin{pmatrix} 0 \\ 0 \\ i4 \end{pmatrix} \quad \dots (5)$$

15

Suppose that the distance between the representative point T of the terminal device and the representative point p1 of the first link e1 is i0, the distance between the representative point p1 of the first link e1 and the representative point p2 of the second link e2 is i1, the distance between the representative point p2 of the second link e2 and the representative point p3 of the third link e3 is i2, the distance between the representative point p3 of the third link e3 and the representative point p4 of the fourth link e4 is i3, the distance between the representative point p4 of the fourth link e4 and the

20  
25

representative point p5 of the fifth link e5 is i4, the distance between the representative point p5 of the fifth link e5 and the representative point p6 of the sixth link e6 is i5, and the distance between the  
 5 representative point p6 of the sixth link e6 and the representative point S of the base is i6.

The representative point p1 of the first link e1 is coincident with a point, on the joint connecting the first link e1 and the second link e2, where the rotation  
 10 axis L1 about which the first link e1 and the second link e2 are turned, and the axis of the first link e1 intersect each other. The second representative point p2 of the second link e2 is on the rotation axis L2 about which the second link e2 and the third link e3 are  
 15 turned and on the joint connecting the links e2 and e3. The representative point p3 of the third link e3 is coincident with a point, on the joint connecting the third link e3 and the fourth link e4, where the rotation axis L3 about which the third link e3 and the fourth  
 20 link e4 are turned, and the axis of the third link e3 intersect each other.

The representative point p4 of the fourth link e4 is coincident with a point, on the joint connecting the fourth link e4 and the fifth link e5, where the rotation  
 25 axis L4 about which the fourth link e4 and the fifth link e5 are turned, and the axis of the fourth link e4 intersect each other. The representative point p5 of the fifth link e5 is coincident with a point, on the joint connecting the fifth link e5 and the sixth link e6,  
 30 where the rotation axis L5 about which the fifth link e5 and the sixth link e6 are turned, and the axis of the fifth link e5 intersect each other. The representative point p6 of the sixth link e6 is coincident with a point, on the joint connecting the sixth link e6 and the base  
 35 22, where the rotation axis L6 about which the sixth link e6 and the base 21 are turned, and the axis of the

sixth link e6 intersect each other.

The position of the triaxial intersection point coincident with the representative point p5 of the fifth link e5 can be determined by using Expression (5). A transformation matrix Ruh for converting the base coordinate system  $\Sigma_{st}$  into the reference coordinate system  $\Sigma_{sb}$  is produced.

$$R_{uh} = R_x[q_x] \cdot R_y[q_y] \cdot R_z[q_z] \cdots (6)$$

10

Values of the coordinates (Pwx, Pwy, Pwz) of the representative point p5 of the fifth link e5 can be calculated by using Expression (7). The coordinates of the representative point p5 of the fifth link e5 are transformed by using Expression (8) for the following operations.

$$p5 = \begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} - R_{uh} \cdot \begin{pmatrix} 0 \\ 0 \\ i5 + i6 \end{pmatrix} = \begin{pmatrix} P_{wx} \\ P_{wy} \\ P_{wz} \end{pmatrix} \cdots (7)$$

20

$$\begin{pmatrix} P_{xn} \\ P_{yn} \\ P_{zn} \end{pmatrix} = \begin{pmatrix} P_{wx} \\ P_{wy} + P_{wz} - i0 \\ P_{wy} - P_{wz} - i0 \end{pmatrix} \cdots (8)$$

An x-coordinate expression Pxn expressing the x-coordinate of the representative point p5 of the fifth link e5 is produced. A yz added value represented by a yz addition coordinate expression Pyn is obtained by adding up the y- and the z-coordinate, and subtracting the distance i0 between the terminal representative point T and the representative point p1 of the first link e1 from the addition of the y- and the z-coordinate. A yz subtraction value represented by a yz subtraction coordinate expression Pzn is obtained by subtracting the

z-coordinate from the y-coordinate, and subtracting the distance between the terminal representative point T and the representative point p1 of the first link e1 from the remainder of subtraction of the z-coordinate from the y-coordinate.

The representative point p5 of the fifth link e5 is coincident with the triaxial intersection point. Therefore, the position of the triaxial intersection point coincident with the representative point p5 can be determined by using Expression (7).

In step s15, the articulated manipulator 20A modeled by the inverted model is transformed into the articulated manipulator 20B modeled by the equivalent model. Rotation angles  $Q_1$ ,  $Q_2$  and  $Q_3$  for the first link e1, the second link e2 and the third link e3 are calculated.

Transformation matrices  $R_{01}$  to  $R_{03}$  in Expression (9) are used for transforming the orientations of the representative points p1 to p3 of the first link e1 to the third link e3 in the reference coordinate system  $\Sigma_{sb}$ . In Expression (9),  $\alpha$  indicates an angle at which the rotation axis L1 about which the first link e1 and the second link e2 turn inclines to the axes of the links e1 and e2. The position of the representative point p5 of the fifth link e5 of the equivalent model shown in Fig. 8 is expressed by Expression (10).

$$\begin{aligned} R_{01} &= R_x [\alpha] \cdot R_z [Q_1] \cdot R_x [-\alpha] \\ R_{02} &= R_{01} \cdot R_z [Q_2] \\ R_{03} &= R_{02} \cdot R_y [Q_3] \end{aligned} \quad \dots (9)$$

$$p5 = R_{01} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + R_{03} \cdot \begin{pmatrix} 0 \\ 0 \\ nr \end{pmatrix} \quad \dots (10)$$

Suppose that the distance  $i_0$  between the terminal representative point T and the representative point  $p_1$  of the first link  $e_1$  is neglected,  $(i_1 + i_2) = 1$ , where  $(i_1 + i_2)$  is the distance between the representative point  $p_1$  of the first link  $e_1$  and the representative point  $p_3$  of the third link  $e_3$ , and  $(i_3 + i_4)/(i_1 + i_2) = nr$ , where  $(i_3 + i_4)$  is the distance between the representative point  $p_3$  of the third link  $e_3$  and the representative point  $p_5$  of the fifth link  $e_5$ .

The actual position of the representative point  $p_5$  of the fifth link  $e_5$  is equal to a value obtained by multiplying a value calculated by using Expression (10) by the distance  $(i_1 + i_2)$  between the representative point  $p_1$  of the first link  $e_1$  and the representative point  $p_3$  of the third link  $e_3$ , and adding the distance  $i_0$  between the terminal representative point T and the representative point  $p_1$  of the first link  $e_1$  to the product of multiplication. The position of the representative point  $p_5$  of the fifth link  $e_5$  determined by using Expression (10) is expressed by Expression (11).

$$p_5 = \begin{pmatrix} P_{5x} \\ P_{5y} \\ P_{5z} \end{pmatrix}$$

$$P_{5x} = -\sin \alpha \cdot \sin Q_1$$

$$+ nr(-\cos Q_3 \cdot \sin \alpha \cdot \sin Q_1 + (\cos Q_1 \cdot \cos Q_2 - \cos \alpha \cdot \sin Q_1) \sin Q_2)$$

$$P_{5y} = -\cos \alpha \cdot \sin \alpha + \cos \alpha \cdot \cos Q_1 \cdot \sin \alpha$$

$$+ nr(\cos Q_3(-\cos \alpha \cdot \sin \alpha \cdot \cos \alpha \cdot \cos Q_1 \cdot \sin \alpha)$$

$$- (\cos \alpha \cdot \cos Q_2 \cdot \sin Q_1 + (\cos^2 \alpha \cdot \cos Q_1 + \sin^2 \alpha) \sin Q_2) \sin Q_3)$$

$$P_{5z} = \cos^2 \alpha + \cos Q_1 \cdot \sin^2 \alpha + nr(\cos Q_3(\cos^2 \alpha + \cos Q_1 \cdot \sin^2 \alpha)$$

$$+ (\cos Q_2 \cdot \sin \alpha \cdot \sin Q_1$$

$$+ (-\cos \alpha \cdot \sin \alpha \cdot \cos \alpha \cdot \cos Q_1 \cdot \sin \alpha) \sin Q_2) \sin Q_3) \quad \dots \quad (11)$$

When the angle  $\alpha$  at which the rotation axis  $L_1$  about which the first link  $e_1$  and the second link  $e_2$  turn inclines to the axes of the links  $e_1$  and  $e_2$  is  $45^\circ$ , Expression (12) is obtained by substituting the angle  $\alpha$

in Expression (11) by  $45^\circ$ .

$$P5 = \begin{bmatrix} -\frac{\sin Q_1}{\sqrt{2}} + nr \left( -\frac{\cos Q_3 \cdot \sin Q_1}{\sqrt{2}} + \left( \cos Q_1 \cdot \cos Q_2 - \frac{\sin Q_1 \cdot \sin Q_2}{\sqrt{2}} \right) \sin Q_3 \right) \\ -\frac{1}{2} + \frac{\cos Q_1}{2} + nr \left( \left( -\frac{1}{2} + \frac{\cos Q_1}{2} \right) \cos Q_3 + \left( \frac{\cos Q_2 \cdot \sin Q_1}{\sqrt{2}} + \left( \frac{1}{2} + \frac{\cos Q_1}{2} \right) \sin Q_2 \right) \sin Q_3 \right) \\ \frac{1}{2} + \frac{\cos Q_1}{2} + nr \left( \left( \frac{1}{2} + \frac{\cos Q_1}{2} \right) \cos Q_3 + \left( \frac{\cos Q_2 \cdot \sin Q_1}{\sqrt{2}} + \left( -\frac{1}{2} + \frac{\cos Q_1}{2} \right) \sin Q_2 \right) \sin Q_3 \right) \end{bmatrix} \dots (12)$$

Expression (13) shows the x-coordinate expression  
 5 Pxn representing the x-coordinate P5x of the  
 representative point p5 of the fifth link e5 in the  
 reference coordinate system, a yz addition coordinate  
 expression Pyn representing the sum of the y- and the z-  
 coordinate of the representative point pt of the fifth  
 10 link e5, and a yz subtraction coordinate expression Pzn  
 representing the remainder of subtraction of the z-  
 coordinate from the y-coordinate of the representative  
 point p5 of the fifth link e5.

$$\begin{aligned} P5x &= Pxn = nr \cdot \cos Q_1 \cdot \cos Q_2 \cdot \sin Q_3 \\ &\quad - \frac{\sin Q_1 (1 + nr \cdot \cos Q_3 + nr \cdot \sin Q_2 \cdot \sin Q_3)}{\sqrt{2}} \\ P5y + P5z &= Pyn \\ 15 \quad &= \sqrt{2} (nr \cdot \cos Q_2 \cdot \sin Q_1 \cdot \sin Q_3 \\ &\quad + \frac{\cos Q_1 (1 + nr \cdot \cos Q_3 + nr \cdot \sin Q_2 \cdot \sin Q_3)}{\sqrt{2}}) \\ P5y - P5z &= Pzn = 1 + nr \cdot \cos Q_3 - nr \cdot \sin Q_2 \cdot \sin Q_3 \quad \dots (13) \end{aligned}$$

The expressions are thus transformed, and the  
 simultaneous equations are solved to produce Expression  
 (14). A third rotation angle  $Q_3$  is calculated by using  
 20 Expression (14). Thus, analytical expressions  
 representing the first rotation angle  $Q_3$ , a second  
 rotation angle  $Q_2$  and third rotation angle  $Q_3$  in the  
 equivalent model can easily be produced.

$$\cos Q_3 = \frac{-2 - 2 \cdot nr^2 + 2 \cdot Pxn^2 + Pyn^2 + Pzn^2}{4 \cdot nr} \quad \dots (14)$$

In step s16, the values of the x- coordinate P5x of the representative point p5 of the fifth link e5, the value of the sum of the y- and the z-coordinate of the representative point p5 of the fifth link e5, and the value of the remainder of subtraction of the z-coordinate from the y-coordinate of the representative point p5 of the fifth link e5 the remainder of subtraction of the z-coordinate from the y-coordinate of the representative point p5 of the fifth link e5 calculated by using Expression (8), namely, the x-coordinate expression Pxn, the yz addition coordinate expression Pyn and the yz subtraction coordinate expression Pzn, in step s14 are substituted into the expressions representing the first rotation angle  $Q_1$  to the third rotation angle  $Q_3$  obtained in step s15 to determine the numerical values of the first rotation angle  $Q_1$  to the third rotation angle  $Q_3$ . Four combinations of those numerical values are possible, provided that the desired position of the base 21 is not a singular point. Step s17 is executed after the numerical values of the first rotation angle  $Q_1$  to the third rotation angle  $Q_3$  in the equivalent model have been determined.

In step s17, the first rotation angle  $Q_1$  to the third rotation angle  $Q_3$  in the equivalent model determined in step s16 are converted into a first rotation angle  $q_1$  to a third rotation angle  $q_3$  in the inverted model. Expression (15) represents  $\cos q_3$ . In Expressions (15), (16) and (17),  $\alpha$  is an angle at which the rotation axis L3 about which the third link e3 and the fourth link e4 turn inclines to the axes of the links e3 and e4.



$$\cos q_3 = (\cos \alpha)^2 \cdot \cos Q_3 - (\cot \alpha)^2 \dots (15)$$

where cosec  $\alpha$  represents the cosecant of the angle  $\alpha$ , and cot  $\alpha$  represents the cotangent of the angle  $\alpha$ . The  
 5 third rotation angle  $q_3$  in the inverted model can be determined by substituting the value of the third rotation angle  $Q_3$  in the equivalent model into Expression (15). Values of  $\cos q_2$  and  $\sin q_2$  are calculated by using Expression (16)

10

$$\begin{aligned} \cos q_2 &= -(\cot \alpha (1 - \cos q_3 \cdot \sin Q_2 + \cos \alpha \cdot \cos Q_2 \cdot \sin q_3) \sin Q_3 / D_2 \\ \sin q_2 &= \cos \alpha (\cos \alpha \cdot \cos Q_2 (1 - \cos Q_3) - \sin Q_2 \cdot \sin Q_3) \sin Q_3 / D_2 \dots (16) \end{aligned}$$

$D_2$  included in Expression (16) is represented by Expression (17).

15

$$D_2 = (\cos \alpha)^2 (1 - \cos q_3)^2 + (\sin q_3)^2 \dots (17)$$

The value of the second rotation angle  $q_2$  in the inverted model can be calculated by using Expression  
 20 (16).

The first rotation angle  $Q_1$  in the equivalent model is equal to the first rotation angle  $q_1$  in the inverted model. Four combinations of the first rotation angle  $q_1$  to the third rotation angle  $q_3$  in the inverted model,  
 25 similarly to those in the equivalent model, provided that the triaxial intersection point is not a singular point.

After the first rotation angle  $q_1$  to the third rotation angle  $q_3$  in the inverted model have been  
 30 determined in step s17, a fourth rotation angle  $q_4$  to a sixth rotation angle  $q_6$  in the inverted model are determined in step s18. A transformation matrix  $R_{wh}$  for the fifth link e5 is represented by Expression (18).

$$R_{wh} = [R_{03}]^t \cdot R_{uh} \quad \dots \quad (18)$$

A transposed matrix obtained by transposing a transformation matrix  $R_{03}$  is multiplied by a transformation matrix  $R_{uh}$  for transforming the base coordinate system  $\Sigma_{st}$  with respect to the reference coordinate system  $\Sigma_{sb}$ . The transformation matrix  $R_{03}$  can be obtained by using Expression (1) because the first rotation angle  $q_1$  to the third rotation angle  $q_3$  are known. The numerical solution of the transformation matrix  $R_{uh}$  can be obtained by using the foregoing expressions. The transformation matrix  $R_{uh}$  can be represented by Expression (6). The fourth rotation angle  $q_4$  to the sixth rotation angle  $q_6$  can be determined by solving Expression (19) when the numerical values of the components of the transformation matrix  $R_{wh}$  are known. In Expression (19),  $\alpha$  is an angle at which the rotation axis  $L_5$  about which the fifth link  $e_5$  and the sixth link  $e_6$  turn inclines to the axes of the links  $e_5$  and  $e_6$ .

$$R_{wh} = R_z[q_4] \cdot R_x[\alpha] \cdot R_z[q_5] \cdot R_x[-\alpha] \cdot R_z[q_6] \quad (19)$$

Two combinations of the fourth rotation angle  $q_4$  to the sixth rotation angle  $q_6$  in the inverted model are possible for each of the combinations of the first rotation angle  $q_1$  to the third rotation angle  $q_3$ , provided that the representative point of the base is not a singular point. Thus, eight combinations of the first rotation angle  $q_1$  to the sixth rotation angle  $q_6$  are possible, provided that the triaxial intersection point and the base representative point are not singular points. Step s19 is executed after the fourth rotation angle  $q_4$  to the sixth rotation angle  $q_6$  in the inverted model have been determined in step s18.

In step s19, the first rotation angle  $q_1$  to the

sixth rotation angle  $q_6$  for the first link e1 to the sixth link e6 in the inverted model are substituted into Expression (20) to convert the first rotation angle  $q_1$  to the sixth rotation angle  $q_6$  into the first rotation angle  $\theta_1$  to the sixth rotation angle  $\theta_6$  for the first link c1 to the sixth link c6 in the true model. After the first rotation angle  $\theta_1$  to the sixth rotation angle  $\theta_6$  have been determined, the positioning data calculating procedure is ended in step s20.

10

$$\begin{aligned}
 \theta_1 &= q_6 \\
 \theta_2 &= q_5 \\
 \theta_3 &= q_4 \\
 \theta_4 &= q_3 \\
 \theta_5 &= q_2 \\
 \theta_6 &= q_1 \\
 &\dots (20)
 \end{aligned}$$

Thus, the first rotation angle  $\theta_1$  to the sixth rotation angle  $\theta_6$  for the links c1 to c6 in the true model can easily be determined by the foregoing positioning data calculating procedure. Thus, the rotation angles through which the links must be turned can be determined to set the terminal device 22 in a desired orientation at a desired position. The links of the articulated manipulator are controlled on the basis of the thus calculated data to set the terminal device 22 in the desired orientation at the desired position. The positioning data calculating procedure may be carried out either by a computer or by an operator.

The thus calculated rotation angles for the links are analytical solutions. Therefore, those rotation angles are accurate and can be calculated in a short time as compared with those obtained by convergence calculation and interpolatory approximation. The rotation angles can be calculated by the positioning

data position procedure by a computer having a low processing ability. The eight combinations of the rotation angles can be obtained, and the optimum one of the eight combinations can selectively determined, considering energy cost and obstructs around the articulated manipulator. The first rotation angle  $\theta_1$  to the sixth rotation angle  $\theta_6$  for the first link c1 to the sixth link c6 of the articulated manipulator 20 shown in Fig. 20 can easily be determined by transforming the true model into the inverted model.

The rotation angles  $\theta_1$  to  $\theta_6$  for the links c1 to c6 in the true model may be determined by producing analytical equations having variables including the distances H0 to H6 between the representative points of the links of the articulated manipulator, the angles  $\alpha$  at which the rotation axes L1 to L6 incline to the axes of the related links, the position T ( $T_x$ ,  $T_y$ ,  $T_z$ ) of the terminal device 22, and the orientation ( $q_0$ ,  $q_a$ ,  $q_t$ ) of the terminal device 22 using the calculated data, and substituting values into the variables of the analytical equations.

In this embodiment, the rotation angles of the links of the articulated manipulator are determined on an assumption that the angles at which the rotation axes incline to the axes of the related links connected by the diagonal joints are  $45^\circ$ . However, the angles at which the rotation axes incline to the axes of the related links connected by the diagonal joints may be different from each other or may be any suitable angles other than  $45^\circ$ .

For example, in the articulated manipulator 20A in the inverted model shown in Fig. 7, the rotation angles  $q_1$  to  $q_6$  for the links e1 to e6 can be determined by multiplying the yz addition coordinate expression  $P_{yn}$  and the yz subtraction coordinate expression  $P_{zn}$  by suitable coefficients, even if the angle at which the

rotation axis  $L1$  about which the first link  $e1$  and the second link  $e2$  turn inclines to the axes of the links  $e1$  and  $e2$  is  $\alpha$ , the angle at which the rotation axis  $L3$  about which the third link  $e3$  and the fourth link  $e4$  turn inclines to the axes of the links  $e3$  and  $e4$  is  $\beta$ , and the angle at which the rotation axis  $L5$  about which the fifth link  $e5$  and the sixth link  $e6$  turn inclines to the axes of the links  $e5$  and  $e6$  is  $\gamma$ . The first to the third rotation angle can more easily be determined when at least one of the rotation axes other than those intersecting at the triaxial intersection point inclines at  $45^\circ$  to the axes of the related links.

Even in a case where the angle at which the rotation axis  $L1$  about which the first link  $e1$  and the second link  $e2$  turn inclines to the axes of the links  $e1$  and  $e2$  is  $45^\circ$ , and the angle at which the rotation axis  $L3$  about which the third link  $e3$  and the fourth link  $e4$  turn inclines to the axes of the links  $e3$  and  $e4$  is not  $45^\circ$  in the inverted model, the first to the third rotation angle can easily be determined by converting the joint connecting the third link  $e3$  and the fourth link  $e4$  into an equivalent model shown in Fig. 8. More concretely, the expression for adding up the  $y$ - and the  $z$ -coordinate of the triaxial intersection point, and the expression for subtracting the  $z$ -coordinate from the  $y$ -coordinate can be simplified, and thereby the first to the third angle can more easily be determined.

Similarly, in a case where the angle at which the rotation axis  $L3$  about which the third link  $e3$  and the fourth link  $e4$  turn inclines to the axes of the links  $e3$  and  $e4$  is  $45^\circ$ , and the angle at which the rotation axis  $L1$  about which the first link  $e1$  and the second link  $e2$  turn inclines to the axes of the links  $e1$  and  $e2$  is not  $45^\circ$ , the first to the third rotation angle can easily be determined by converting the joint connecting the first link  $e1$  and the second link  $e2$  into an equivalent model.

When the articulated manipulator has at least the rotation axis about which the two links connected by the diagonal joint among the joints connecting the first to the third link turn inclines at an angle of  $45^\circ$  to the axes of the two links connected by the same diagonal joint, the rotation angles for the links can more simply be determined. The triaxial intersection point in this embodiment is on the base in the true model, the triaxial intersection point may be on the terminal device. If the triaxial intersection point is on the terminal device, the rotation angles for the link can easily be determined without requiring the transformation of the true model into the inverted model.

As shown in the true model, in a first articulated manipulator in which rotation axes L1, L2 and L3 near a fixed part intersect each other at a triaxial intersection point, a rotation axis L3 about which third and fourth links turn inclines at  $45^\circ$  to the axes of the third and the fourth link, and a rotation axis L6 about which fifth and sixth links turn inclines at an optional angle to the axes of the fifth and the sixth link, or in a second articulated manipulator in which rotation axes L1, L2 and L3 near a fixed part intersect each other at a triaxial intersection point, the rotation axis L3 about which third and fourth links turn inclines at an optional angle to the axes of the third and the fourth link, and a rotation axis L6 about which fifth and sixth links turn inclines at  $45^\circ$  to the axes of the fifth and the sixth link, rotation angles for the links can more easily be determined by transforming a true model into an inverted model, and converting the joint having the rotation axis inclined at the optional angle to the axes of the related links into an equivalent model.

As shown in the inverted model, in a third articulated manipulator in which rotation axes L4, L5 and L6 near a free part intersect each other at a

triaxial intersection point, a rotation axis L1 about  
 which first and second links turn inclines at  $45^\circ$  to the  
 axes of the first and the second link, and a rotation  
 axis L3 about which third and fourth links turn inclines  
 5 at an optional angle to the axes of the third and the  
 fourth link, or in a fourth articulated manipulator in  
 which rotation axes L4, L5 and L6 near a free part  
 intersect each other at a triaxial intersection point,  
 the rotation axis L1 about which first and second links  
 10 turn inclines at an optional angle to the axes of the  
 first and the second link, and a rotation axis L3 about  
 which third and fourth links turn inclines at  $45^\circ$  to the  
 axes of the third and the fourth link, rotation angles  
 for the links can easily be determined by converting the  
 15 joint having the rotation axis inclined at the optional  
 angle to the axes of the related links into an  
 equivalent model.

The rotation angles for the links can be determined  
 even if the rotation axis of the joint connecting the  
 20 third link e3 and the fourth link e4 is orthogonal to  
 the axes of the links e3 and e4 as shown in Fig. 8.

Fig. 9 shows a model modeling an articulated  
 manipulator in a second embodiment according to the  
 present invention. In the model shown in Fig. 9, a sixth  
 25 link e6 and a base 21 corresponding to those of the  
 inverted model shown in Fig. 7 are fixedly joined  
 together. Rotation angles  $q_1$  to  $q_5$  for a first link e1  
 to a fifth link e5 can be determined by the aforesaid  
 positioning data calculating procedure.

30 As apparent from the foregoing description,  
 according to the present invention, the first to the  
 sixth angles for locating the object in a desired  
 orientation at a desired position can be determined in a  
 short time. Thus, the rotation angles through which the  
 35 links are to be turned can be determined every time the  
 object is moved. For example, data on the orientation

and position of the object moving along a prescribed path is entered continuously to change the angular position of the links can sequentially be changed on the basis of the continuously entered data. Thus, the object  
5 can quickly and smoothly be moved to the desired position.

The values of the rotation angles can analytically be determined, and all the possible combinations of the rotation angles can be obtained. In a six-degree of  
10 freedom articulated manipulator, eight combinations of rotation angles can be obtained. Optimum one of the eight combinations of the rotation angles for the links of the articulated manipulator can be chosen to locate the object in a desired orientation at a desired  
15 position. For example, optimum one of the eight combinations of the rotation angles is selected taking into consideration energy cost and obstacles around the articulated manipulator.

According to the present invention, the first to  
20 the fifth rotation angles for locating the object in a desired orientation at a desired position can be determined in a short time. Thus, rotation angles for the links can be determined every time the object is moved.. For example, data on the orientation and  
25 position of the object moving along a prescribed path is entered continuously to change the angular position of the links can sequentially be changed on the basis of the continuously entered data. Thus, the object can quickly and smoothly be moved to the desired position.

30 The values of the rotation angles can analytically be determined, and all the possible combinations of the rotation angles for moving the object to the desired position can be obtained. One of the combinations of the rotation angles for the links of the articulated  
35 manipulator can be chosen to locate the object in a desired orientation at a desired position. For example,



optimum one of the combinations of the rotation angles is selected taking into consideration energy cost and obstacles around the articulated manipulator.

According to the present invention, the calculating  
5 device is capable of quickly determining the first to the sixth rotation angles to locate the object in a desired orientation at a desired position. Thus, the rotation angles through which the links are to be turned can be determined every time the object is moved. The  
10 output device sends the calculated data to the articulated manipulator. For example, the input device enters data on the orientation and position of the object moving along a prescribed path continuously, and the links are turned through the rotation angles  
15 provided by the output device, and thereby the angular position of the links can sequentially be changed on the basis of the continuously entered data. Thus, the object can quickly and smoothly be moved to the desired position.

20 The calculating device determines values of the rotation angles analytically, and all the possible combinations of the rotation angles to move the object to the desired position can be obtained. For example, in a six-degree of freedom articulated manipulator, eight  
25 combinations of rotation angles can be obtained. One of the combinations of the rotation angles for the links of the articulated manipulator can be chosen to locate the object in a desired orientation at a desired position. For example, optimum one of the combinations of the  
30 rotation angles is selected taking into consideration energy cost and obstacles around the articulated manipulator.

According to the present invention, the calculating  
35 device is capable of quickly determining the first to the fifth rotation angles to locate the object in a desired orientation at a desired position. Thus, the

rotation angles through which the links are to be turned can be determined every time the object is moved. The output device sends the calculated data to the articulated manipulator. For example, the input device  
5 enters data on the orientation and position of the object moving along a prescribed path continuously, and the links are turned through the rotation angles provided by the output device, and thereby the angular position of the links can sequentially be changed on the  
10 basis of the continuously entered data. Thus, the object can quickly and smoothly be moved to the desired position.

The calculating device determines values of the rotation angles, and all the possible combinations of  
15 the rotation angles can be obtained. One of the combinations of the rotation angles for the links of the articulated manipulator can be chosen to locate the object in a desired orientation at a desired position. For example, optimum one of the combinations of the  
20 rotation angles is selected taking into consideration energy cost and obstacles around the articulated manipulator.

According to the present invention, the rotation axes about which the object and the sixth link turn, the  
25 sixth and the fifth link turn, and the fifth and the fourth link turn, respectively, are coincident at the triaxial intersection point. Therefore the coordinates of the triaxial intersection point can uniquely be determined by an orientation controller when the  
30 position and orientation of the object is determined. Thus, the orientation controller is able to determine analytically the first to the sixth rotation angle for the links for locating the object in a desired orientation at a desired position. Whereas the  
35 conventional articulated manipulator provided with six joints is incapable of analytically determining the

rotation angles, the articulated manipulator of the present invention is capable of analytically determining the rotation angles. Consequently, the object can quickly and accurately be located.

5           When the rotation axis about which the two links connected by at least the diagonal joints among the joints connecting the first to the third link turn inclines at an angle of  $45^\circ$  to the axes of the links connected by the same diagonal joint, the orientation  
10 controller is able to determine the rotation angles by using simplified expressions. Consequently, time necessary for the orientation controller to determine the first to the sixth rotation angles can further shortened.

15           According to the present invention, the rotation axes about which the object and the sixth link turn, the sixth and the fifth link turn, and the fifth and the fourth link turn, respectively, are coincident at the triaxial intersection point. Therefore the coordinates  
20 of the triaxial intersection point can uniquely be determined by an orientation controller when the position and orientation of the object is determined. Thus, the orientation controller is able to determine analytically the first to the sixth rotation angle for  
25 the links for locating the object in a desired orientation at a desired position.

          When the rotation axis about which the two links connected by at least the diagonal joints among the joints connecting the first to the third link turn  
30 inclines at an angle of  $45^\circ$  to the axes of the links connected by the same diagonal joint, the orientation controller is able to determine the rotation angles by using simplified expressions. Consequently, time necessary for the orientation controller to determine  
35 the first to the sixth rotation angles can further shortened. Whereas the conventional articulated

manipulator provided with five joints is incapable of analytically determining the rotation angles, the articulated manipulator of the present invention is capable of analytically determining the rotation angles.

5 Consequently, the object can quickly and accurately be located.

Although the invention has been described in its preferred embodiments with a certain degree of particularity, obviously many changes and variations are  
10 possible therein. It is therefore to be understood that the present invention may be practiced otherwise than as specifically described herein without departing from the scope and spirit thereof.